A BRIEF NOTE ON TOLL ROAD ASSIGNMENT METHODS

Draft Working Paper
July 2013, Revised September 2014

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Introduction

In this note, we examine a simple, hypothetical toll road assignment problem and show how it can illuminate some of the issues in toll road modeling. We consider only static user equilibrium methods and static toll levels, despite our preference for dynamic analysis. Nevertheless, one simple example illustrates the intuitive point that the distribution of value of time among travelers is the key determinant of toll facility utilization. We also show that different assignment methods and implementation practices lead to significant variations in predictions of toll road use and the associated revenue.

Current Traffic Assignment Methods and Practices

There are several basic approaches to toll road assignment commonly encountered in practice. Some form of deterministic or stochastic user equilibrium assignment has become recognized as essential because it provides a reasonable behavioral paradigm. It also serves the need to achieve consistency in model inputs and outputs either solely in the traffic assignment model or in a model sequence that culminates with a traffic assignment and has feedback to earlier stages of the model influencing trip distribution, mode choice, or both of these model components. The time of day of travel may also be influenced by travel times and costs that differ by time of day.

In this note, we consider only the traffic assignment aspects and extensions. Mode choice, time-of – day of travel, and other more advanced demand considerations are discussed in NCHRP 722 and widely in the research literature. Of course, most of these other model components would be impaired by faulty traffic assignments and incorrect congested travel times.

The basic variation introduced by accounting for toll roads in traffic assignment is that a price is to be paid for a faster and potentially more reliable route. This price or toll may vary by vehicle class, and classes may differ in the value they place on time in different ways than are reflected in the toll. Tolls may be link-based such as a toll for a bridge crossing, or may be more complex, as in the case of entrance-to-exit tolls that cannot be represented correctly by link tolls.

Conventional link-based traffic assignment algorithms can account for ramp-to-ramp or entrance-to-exit tolls but not without some methodological extension. TransCAD supports these tolls in addition to link tolls (Caliper 2012). Note that entrance-to-exit tolls are more varied than origin-to-destination (O-D) tolls because different routes for a given O-D pair can use different toll road entrance and exit ramps.

Standard, good practice for toll road modeling is to use a multi-class assignment that respects link class exclusions, represents user classes with different mean values of time, and accurately represents the tolls that are applicable. This method is recommended in NCHRP 722, although it is an open question as to how many different user classes should be used.

A different method that is in use, particularly in "investment grade" studies and also in road pricing studies, is a legacy method that is based upon computing the best toll path and the best non-toll path at each iteration of the assignment calculation. A logit model or an alternative method such as a look-up table is then applied to split the O-D traffic loaded between the two paths based on the toll, the time differential between the best toll and non-toll paths, and possibly other factors (URS 2011). This is commonly called the "toll diversion" method. As we will see later, this method is flawed and is not the best available approach.

More complex methods consider value of time distributions explicitly, but have been rarely used in practice. Leurent (1993) proposed a time and cost assignment with a value of time distribution but with only time being volume-dependent. Caliper successfully implemented this method for SETRA, the French Ministry of Transport where it is used for modeling toll roads. Leurent's method is solved by the method of successive averages (MSA). Dial (1997) introduced a bi-criterion traffic assignment he called T2 in which both time and cost or another dimension could be volume-dependent. When applied to toll road assignment, a stochastic value of time distribution is used. We have experimented with T2 and have implemented several versions of T2 within the context of our multi-modal, multi-class master assignment procedure. A multi-modal, multi-class method is needed to account for network exclusions for certain classes of traffic.

A Simple Example for Testing Equilibrium Toll Road Assignment Methods

To investigate toll road assignment methods, and to illustrate the extreme differences in results that are associated with different traffic assignment approaches for toll roads, we will present a small hypothetical example for a single origin-destination pair. The network is shown in Figure 1. There are two toll links that provide short free flow travel times of 10 minutes and other links that have free flow travel times of 20 minutes each. As shown in Figure 2, the first and last links have capacities of 200 vehicles per hour which will not restrict the flow of the 100 trips that will be assigned. The interior links all have capacities of 50 vehicles per hour.

To reach the destination from the origin, the links in this network can be traversed in three ways as illustrated in Figure 3. In the first, no toll links are utilized. In the second path, only the less expensive toll link is utilized, and in the third path, both toll links are traversed.

Ordinarily we would concern ourselves with the degree of convergence achieved and the solution algorithm employed. Due to the simplicity of this problem, all of the assignments we present are fully converged, and the path volumes are uniquely determined, which would not be so in the more general case of multiple origins and destinations.

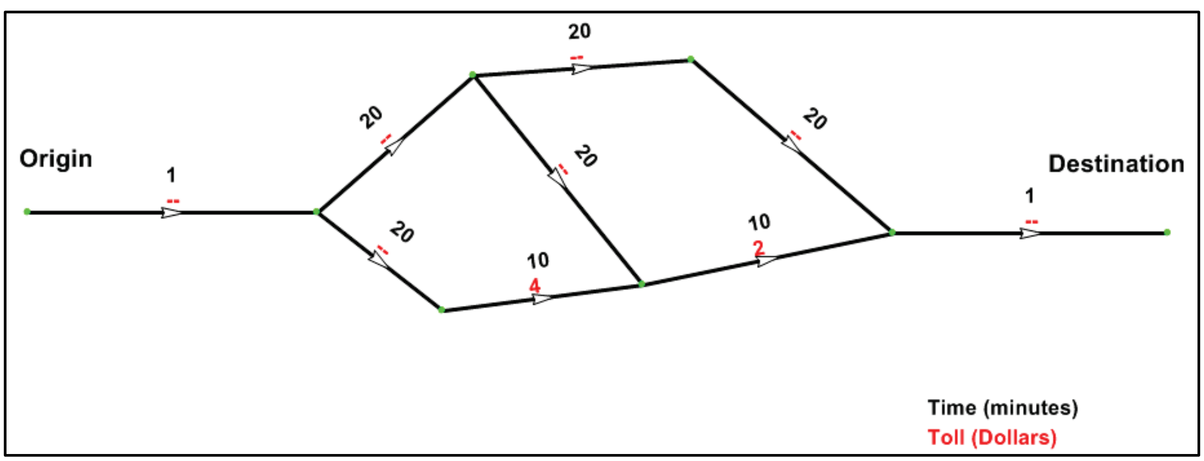

Figure 1 Network Travel Times and Tolls

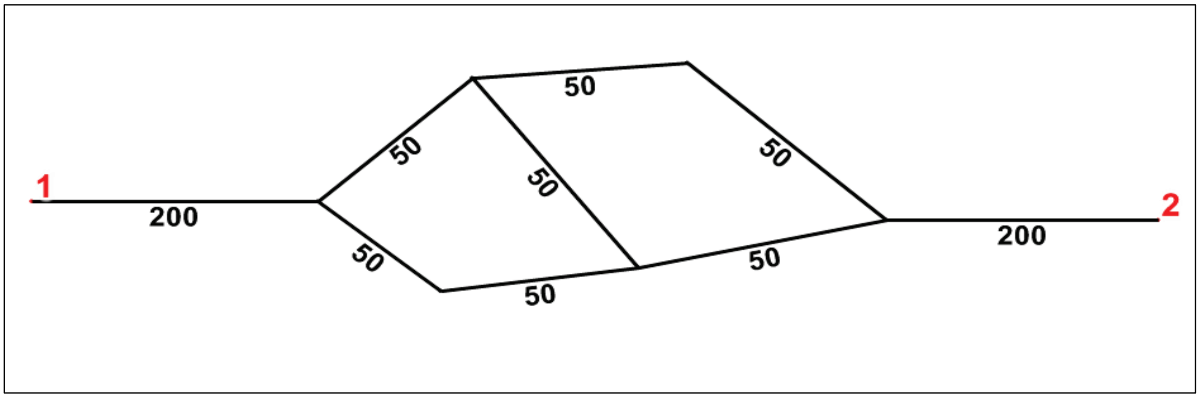

Figure 2 Link Capacities and O-D Demand

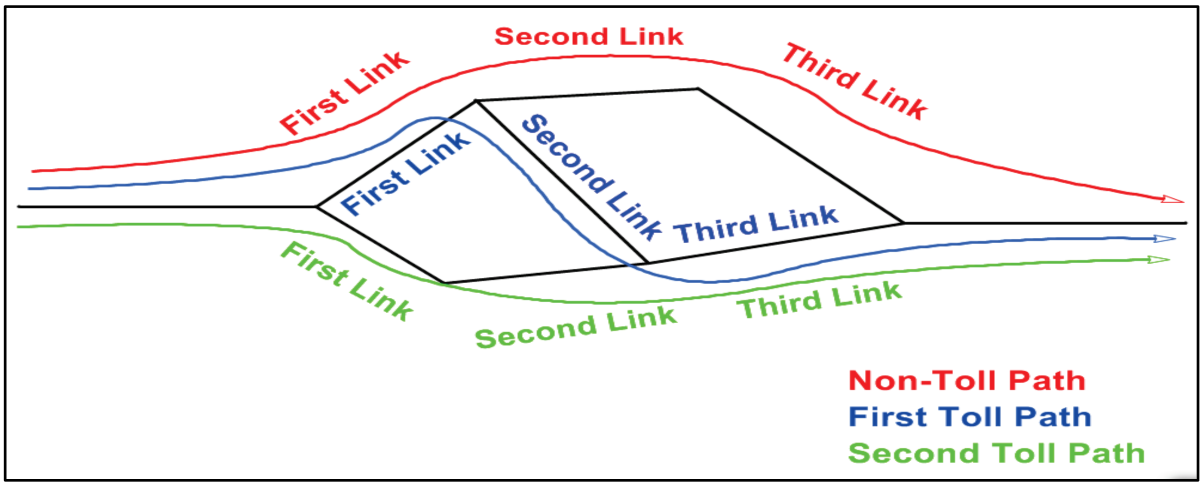

Figure 3 Possible Network Paths

Single Class User Equilibrium Results

As a point of reference, we first compute a single class user equilibrium solution to the problem posed above utilizing a generalized cost function computed with a value of time (VOT) of $20 per hour. The results are shown below in terms of the link loadings.

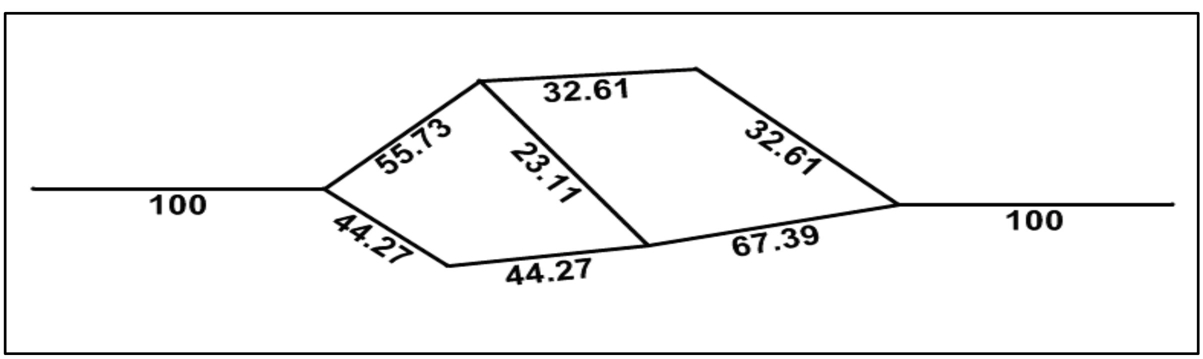

Figure 4 Single Class User Equilibrium Link Flows

SINGLE CLASS USER EQUILIBRIUM RESULTS

	Non-toll path	First toll path	Second toll path
Link 1 time	24.63	24.63	21.84
Link 2 time	20.54	20.14	10.92
Link 3 time	20.54	14.95	14.95
Total time(minutes)	65.71	59.72	47.71
Volume	**32.61**	**23.11**	**44.27**
Path Gen. Cost(min.)@ $20 VOT	65.71	65.72	65.71
Path Toll / Time Equiv.	0 , 0	$2 , 6 min.	$6 , 18 min.
Toll Revenue	0	$46.22	$381.42

Total VHT = 5635.054	Total Toll Revenue = $427.64

Table 1 Single Class User Equilibrium Results

At user equilibrium, all used paths have the same generalized cost. One can see that the equilibrium condition is met as the generalized cost is the same 65.7 minutes for each path.

Toll Diversion Method

In contrast, we perform a toll diversion assignment on the same network using the same $20/hour mean value of time. The specific logit equation is given below:

- $U_{Toll} = C + \alpha_1(Time_{Toll} - Time_{Free}) + \alpha_2(Cost_{Toll} - Cost_{Free}) + \alpha_3(Dist_{Toll} - Dist_{Free})$
- $P_{Toll} = \frac{1}{1+e^{-U_{Toll}}}$

In the example above,

- $C = 0$
- $\alpha_1 = \; -1.0$
- $\alpha_2 = \; -3.0$
- $\alpha_3 = 0$
- VOT = $\frac{\alpha_1}{\alpha_2}$ = 0.33$/min = $20/hr

This equation has been used by consultants in other analyses. Also, one may encounter a similar approach with a "willingness-to-pay" look-up table that provides a set of toll path choice probabilities given toll costs and time savings.

In the toll diversion method, toll values are not used in finding the best toll path, which is a problem when there is more than one toll path for a single origin-destination pair. Typically, the toll values also are not part of the relative gap calculation used as a stopping criterion for the equilibrium calculation. This means that the solution is different from the one in which there is an equilibrium in the generalized cost domain.

The results for the toll diversion assignment are shown in Table 2 below. The flow pattern is particularly unusual with two thirds of travelers using the toll road but only 3.4 choosing the first toll path.

TOLL DIVERSION ASSIGNMENT RESULTS

	Non-toll path	First toll path	Second toll path
Link 1 time	20.85	20.85	27.84
Link 2 time	20.57	20.00	13.92
Link 3 time	20.57	14.83	14.83
Total time (minutes)	61.99	55.68	56.59
Volume	**33.03**	**3.4**	**63.57**
Path Gen. Cost (min.)@ $20 VOT	61.99	61.68	74.59
Path Toll / Time Equiv.	0 , 0	$2 , 6 min.	$6 , 18 min.
Toll Revenue	0	6.8	381.42

Total VHT = 5834.27 min.	Total Toll Revenue = 388.22

Table 2 Toll Diversion Assignment

In this assignment, only 3.4 trips are assigned to the first toll path, which seems unrealistic. As can be observed from these results, a user equilibrium solution is not achieved, as evidenced by the radically different generalized costs associated with the different paths. Moreover, the highest volume path has the highest generalized cost, and the path with the lowest generalized cost attracts the fewest trips. Clearly, the Wardrop equilibrium condition that all used paths have the same minimum generalized cost is not satisfied because it would appear that travelers could shift from the third path to either of the other two and experience lower generalized costs.

Clearly, one of the problems with the toll diversion approach is that the pathfinding ignores the tolls while the loading is based upon the tolls and travel times. This is an inconsistency that magnifies the problems with toll diversion and certainly breaks the concept of a user equilibrium solution.

Note that the toll diversion model “converges”. It just converges to a solution that is not the generalized cost equilibrium solution.

A misconception in the toll diversion method is that there is inherently a distribution of value of time and not just a single value. Rather, there is a distribution of toll path choice probabilities as a function of toll costs per travel time savings. Moreover, the route choice is not actually implemented as a probability but as a share model. Of course, both the value of time and other toll choice determinants can vary by origin and/or destination. But when there is only one O-D pair, that cannot be done, and the shortcomings of the method are revealed. Even if there are separate equations and VOTs for different trip purposes or other traveler classifications, each market segment assignment will have the problems identified above. Practitioners of the toll diversion method may often use constants to control the share of toll road users by O-D pair. This should be regarded as an ad hoc off-model adjustment unless it is strictly based upon empirical measurements.

An alternative toll diversion heuristic was proposed by Florian (2006). This heuristic has somewhat different properties than the method described above, but also fails to provide reasonable results on this simple example.

It is interesting to observe how different the estimated VHT and toll revenue from toll diversion are when compared to the standard UE assignment. VHT is over-estimated and toll revenue is under-estimated in this example. One might speculate that this provides a possible explanation for unduly optimistic toll road forecasts. Of course, for other examples or other parameters, different behaviors might be observed depending upon free-flow speeds and toll levels.

Multi-class User Equilibrium Assignment

While the regular UE solution is clearly superior to the toll-diversion method, it is also an unacceptable simplification of reality in that different travelers have different values of time. The effects can be seen from computing a multi-class traffic assignment in which the classes are segmented by different values of time.

First, we assume a uniform distribution of VOT with 5 user classes and the same mean VOT of $20/hour for all travelers as in the prior examples. The classes have VOTs of 0, 10, 20, 30, and 40 dollars per hour, respectively.

MULTI-CLASS TRAFFIC ASSIGNMENT WITH 5 USER CLASSES AND UNIFORM VOT DISTRIBUTION

	Non-toll path	First toll path	Second toll path
Link 1 time	24.69	24.69	21.81
Link 2 time	21.23	20.03	10.91
Link 3 time	21.23	13.1	13.1
Total time (minutes)	67.15	57.82	45.82
Volume	**40**	**15.91**	**44.08**
Toll Revenue	0	31.82	264.48

Total VHT = 5625.66 min.	Total Toll Revenue = $296.3

Table 3 Multi-class Traffic Assignment with 5 User classes and Uniform VOT distribution

Here we have slightly lower VHT and also lower toll revenue than for the single class UE, presumably because there are more travelers with low values of time.

It is generally thought that a more realistic value of time distribution is lognormal. For purposes of illustration, we use a lognormal distribution with same mean value of time of $20 and a standard deviation of $20. The percentage of travelers by value of time is shown in the graph below and compared to the uniform distribution. It has a wider dispersion in that there are some travellers with a higher value of time than $40 per hour but also many more with low values of time.

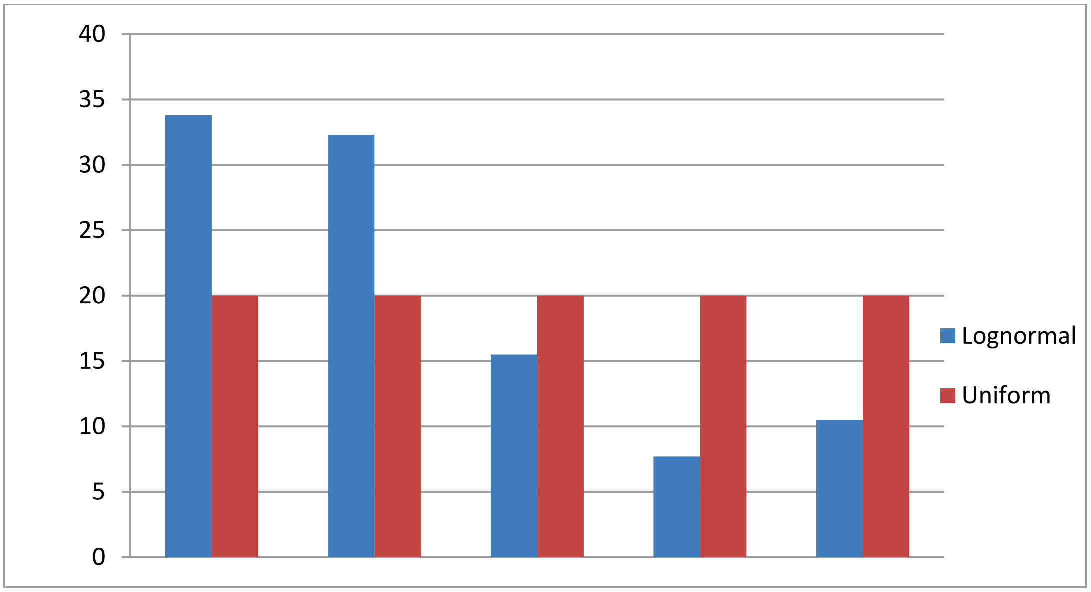

Figure 5 Comparison of Lognormal and Uniform Value of Time Distributions

A multi-class assignment with the same number of classes but with the lognormal distribution of class volumes by value of time is shown below.

	Non-toll path	First toll path	Second toll path
Link 1 time	29.19	29.19	20.63
Link 2 time	22.09	20.08	10.32
Link 3 time	22.09	12.09	12.09
Total time	73.37	61.36	43.04
Volume	**45.67**	**20.47**	**33.86**
Toll Revenue	0	40.94	203.16

Total VHT = 6064.18 min.	Total Toll Revenue = $244.1

Table 4 User Equilibrium Assignment with 5 User Classes and Lognormal VOT Distribution (0,10,20,30,40$/hr VOT classes)

In the table above, we observe higher VHT and lower toll revenue that in the uniform VOT, multi-class assignment with a single user class and the same mean value of time. Clearly, the VOT distribution has a significant effect on the assignment results.

A more refined calculation of the UE assignment comes from more closely approximating the assumed value of time distribution. This can be done by using more finely-grained value of time classes for the multi-class assignment. The results for a 20-class assignment are shown below.

MULTI-CLASS USER EQUILIBRIUM WITH A LOGNORMAL VOT DISTRIBUTION AND 20 USER CLASSES

	Non-toll path	First toll path	Second toll path
Link 1 time	26.62	26.62	21.12
Link 2 time	21.52	20.06	10.56
Link 3 time	21.52	12.68	12.68
Total time	69.66	59.36	44.36
Volume	**42.18**	**18.75**	**39.07**
Toll Revenue	0	$38	$234

Total VHT = 5784 min.	Total Toll Revenue = $272

Table 5 Multi-class User Equilibrium with a Lognormal VOT Distribution and 20 User Classes

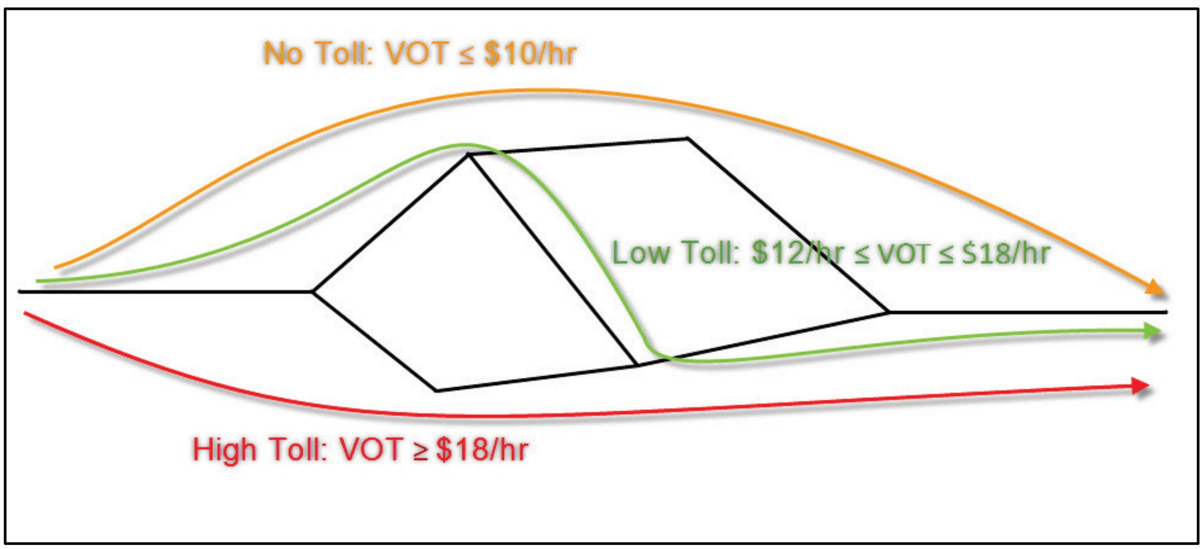

Figure 6 Paths Chosen by Value of Time Class in 20 Class MMA Assignment

When compared to the 5-class solution, there is a more than a 10% difference in toll revenues in the 20-class solution.

To carry this process further, we present a 100 class assignment below. Here the results are similar, but there are still some differences. This illustrates that there is sensitivity to closeness with which the VOT distribution is approximated with discrete user classes.

MULTI-CLASS USER EQUILIBRIUM WITH A LOGNORMAL VOT DISTRIBUTION AND 100 USER CLASSES

	Non-toll path	First toll path	Second toll path
Link 1 time	26.01	26.01	21.29
Link 2 time	21.52	20.04	10.65

Link 3 time	21.52	12.68	12.68
Total time	69.05	58.73	44.62
Volume	**42.18**	**17.31**	**40.51**
Toll Revenue	0	$35	$243

Total VHT = 5737	Total Toll Revenue = $278

Table 6 Multi-class User Equilibrium with a Lognormal VOT Distribution and 100 User Classes

There is a progression of flow estimates for the second toll path which seems to be moving toward roughly a 17 % market share. We could have continued to fractionate the trip table further, but stopped at this level in which the number of classes is equal to the number of trips.

The fact that this approach appears to provide reasonable answers does not address a variety of issues with large scale multi-class assignments such as the non-uniqueness of class flows and path flows and the consequences that may hold for toll revenue estimation (Engelson and Lindberg, 2006) or problems in validating multi-class models with observed data. In the future, we will discuss these issues more directly.

In further work, we also will compare the multi-class approach with a large number of user classes with a bi-criterion assignment with the same value of time distribution. We would expect to obtain similar results, but this remains to be seen. Also, the computational intensity of both approaches should be compared as this is likely to be a factor in the selection of approaches for large regional models.

Concluding Remarks

The results for the simple, notional example are only likely to be exacerbated in real, large traffic assignment models, which bring numerous additional complexities to toll road analysis. These include problems with convergence, the non-uniqueness of user class flows (and thus toll revenues), and accurate representation of behavioral and traffic dynamics. The sensitivity of the toll road assignments to both VOT distributions and the choice of assignment approach add to these difficulties in formulating more complex, region-traffic assignment models for evaluating toll facilities and other pricing initiatives.

Since VOT distributions are not directly observable, they must be estimated for specific populations from stated preference experiments. These measurements, no matter how well-executed, bring their own vagaries and errors, which when taken with the sensitivity of assignments to VOT distributions, must render most toll road estimates rather uncertain. VOT distribution estimates are more likely to be representative for large samples of travelers than by origin-destination pair or other market segment, which may have other ramifications for the reliability of forecasts.

Given a choice of methods, it would seem that the toll diversion approach is inappropriate for use in nearly any context, and that multi-class UE methods or more elaborate bicriterion and dynamic methods would be preferable. With better methods, analysts should expect considerable sensitivity to the value of time distribution and further but hopefully lesser sensitivities to the assignment method and the details of its implementation. We will be continuing to research more appropriate methods for analyzing pricing in large regional models and hope to provide better methods for future use.

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